



Roll No. _____ to be filled in by the candidate

(For All Sessions)

Time: 30 Minutes

Marks: 20

Mathematics (Objective) *Lwp-12-1-23*

(Group-I)

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

- 1.1 $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = ?$ (A) 1 (B) 2 (C) 0 (D) -2
2. $\cos hx + \sin hx = ?$ (A) e^x (B) e^{-x} (C) e^{2x} (D) $2e^x$
3. $\frac{d}{dx} [\ln(2^x)] = ?$ (A) $\ln 2$ (B) 2^x (C) $\frac{1}{2^x}$ (D) $\frac{\ln 2}{2^x}$
4. $\frac{d}{dx} (\cos hx) = ?$ (A) $-\sin hx$ (B) $\sin hx$ (C) $\operatorname{sech} hx$ (D) $\operatorname{cosech} hx$
5. If $f(x) = \sqrt{x}$, then $f^{-1}(0) = ?$ (A) 0 (B) 1 (C) Undefined (D) $\frac{1}{2}$
6. $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = ?$ (A) 1 (B) 0 (C) -1 (D) 2
7. $\int dx = ?$ (A) x (B) $\frac{1}{x}$ (C) x^2 (D) $\frac{1}{x^2}$
8. $\int e^x(x+1) dx = ?$ (A) $xe^x + c$ (B) $e^x + c$ (C) $x + c$ (D) $x^2 + c$
9. $\int_0^{\pi/2} \cos x dx = ?$ (A) 0 (B) -1 (C) 1 (D) 2
10. $\int \frac{\sin 2x}{\sin x} dx = ?$ (A) $2 \cos x + c$ (B) $2 \sin x + c$ (C) $\frac{1}{2} \sin x + c$ (D) $\frac{1}{2} \cos x + c$
11. The slope of a line $x = 5$ is: (A) 0 (B) 1 (C) -1 (D) Infinite
12. Midpoint of (0, -2) and (-2, 0) is: (A) (0, 0) (B) (-1, -1) (C) (-2, -2) (D) (0, -1)
13. Distance between (-1, 2) & (7, 5) is: (A) $\sqrt{73}$ (B) 7 (C) $2\sqrt{73}$ (D) 73
14. The solution of inequality $x + 2y < 6$ is: (A) (1, 4) (B) (1, 3) (C) (1, 1) (D) (1, 5)
15. Equation of Tangent to $x^2 + y^2 = 4$ at (2, 0) is: (A) $x = 1$ (B) $y = 1$ (C) $y = 2$ (D) $x = 2$
16. Slope of tangent to parabola $y^2 = 4ax$ at (a, 2a) is: (A) 2 (B) -1 (C) 1 (D) 3
17. Eccentricity e of a circle is: (A) $e = 0$ (B) $e = 1$ (C) $0 < e < 1$ (D) $e > 1$
18. Radius of a circle $x^2 + y^2 = 2$ is: (A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\sqrt{2}$
19. If $P = (2, 3)$, $Q = (6, -2)$, then $|\overline{PQ}|$ is: (A) $\sqrt{40}$ (B) $\sqrt{42}$ (C) $\sqrt{41}$ (D) $\sqrt{43}$
20. For a vector $\vec{v} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, then $\cos \beta = ?$ (A) $\frac{3}{7}$ (B) $\frac{2}{7}$ (C) $-\frac{6}{7}$ (D) $-\frac{3}{7}$

Mathematics (Subjective)

(For All Sessions)
(GROUP-I)

Time: 2:30 hours

SECTION-I

Rwp-12-1-23

2. Write short answers of any eight parts from the following: (8x2=16)

- Express perimeter P of a square as a function of its area A .
- Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$
- Define even function with example.
- Find derivative by definition $\frac{1}{\sqrt{x}}$
- If $y = x^4 + 2x^2 + 2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$
- Differentiate w.r.t x , $y = x^2 \sec 4x$
- Find $\frac{dy}{dx}$; $xy + y^2 = 2$
- Find $\frac{dy}{dx}$ if $y = x\sqrt{\ln x}$
- Differentiate w.r.t x , $y = \cot^{-1}\left(\frac{x}{a}\right)$
- Apply the Maclaurin series to prove that: $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \dots$
- Graph the solution set of $2x + y \leq 6$.
- Define feasible region.

3. Write short answers of any eight parts from the following: (8x2=16)

- Evaluate $\int \tan^2 x dx$.
- Evaluate $\int \frac{(a-b)x}{(x-a)(x-b)} dx$
- Evaluate $\int x \sin x dx$.
- Evaluate $\int_{\pi/6}^{\pi/3} \cos t dt$
- Solve the differential equation $y dx + x dy = 0$
- Evaluate $\int \frac{x^2}{4+x^2} dx$
- Find the areas between the $x - ax$ and the curve $y = x^2 + 1$ from, $x = 1$ to $x = 2$
- Find direction cosines of $\underline{V} = 4\underline{i} - 5\underline{j}$
- Find a unit vector in the direction of $\underline{V} = \frac{4}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$
- Find α , so that vector $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$, $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$ are perpendicular.
- Find the area of parallelogram whose vertices are: $A(0, 0, 0)$ $B(1, 2, 3)$ $C(2, -1, 1)$ $D(3, 1, 4)$
- A force $\underline{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$ is applied at $p(1, -2, 3)$. Find its amount about the point $Q(2, 1, 1)$

4. Write short answers of any nine parts from the following: (9x2=18)

- Is $(\sqrt{176}, 7)$ at a distance of 15 units from the origin?
- By means of slopes, show that the point $(-4, 6)$, $(3, 8)$, $(10, 10)$ lie on the same line.
- Find K so that the line joining $A(7, 3)$; $B(k, -6)$ and the line joining $C(-4, 5)$, $D(-6, 4)$ are parallel.
- Find the equation of the line having y -intercept -7 and slope -5 .
- Find the point of intersection of the lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$
- Find equation of lines represented by $2x^2 + 3xy - 5y^2 = 0$
- Find the measure of the angle between the lines represented by $9x^2 + 24xy + 16y^2 = 0$
- Find an equation of the circle with ends of diameter at $(-3, 2)$ and $(5, -6)$
- Show that the line $2x + 3y - 13 = 0$ is tangent to the circle $x^2 + y^2 + 6x - 4y = 0$
- Check the position of the point $(5, 6)$ with respect to the circle $x^2 + y^2 = 81$.
- Find focus and directrix of the parabola $x^2 = -16y$
- Find an equation of ellipse if foci $(-3\sqrt{3}, 0)$ and vertices $(\pm 6, 0)$.
- Find equation of hyperbola with given data foci $(0, \pm 9)$, directrices $y = \pm 4$

SECTION-II

Note Attempt any three questions. Each question carries equal marks: (10x3=30)

- (a) Evaluate: $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$ (b) If $y = \tan\left(2 \tan^{-1} \frac{x}{2}\right)$, then show that $\frac{dy}{dx} = 4\left(\frac{1+y^2}{4+x^2}\right)$
- (a) Evaluate: $\int \frac{dx}{\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x}$ (b) Find equation of line through intersection of $x + 2y + 3 = 0$, $3x + 4y + 7 = 0$ and making equal intercepts on the axes.
- (a) Find the area bounded by the curve $f(x) = x^3 - 2x^2 + 1$ and x -axis in the 1st quadrant.
(b) Minimize $Z = 3x + y$ subject to the constraints $3x + 5y \geq 15$ $x + 6y \geq 9$ $x \geq 0$, $y \geq 0$
- (a) If $y = a \cos(\ln x) + b \sin(\ln x)$ prove that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$
(b) Find the coordinates of the points of intersection of the line $2x + y = 5$ and the circle $x^2 + y^2 + 2x - 9 = 0$, also find the length of intercepted chord.
- (a) Find the centre foci, eccentricity and vertices of the ellipse $x^2 + 16x + 4y^2 - 16y + 76 = 0$



Roll No. _____ to be filled in by the candidate

HSSC-(P-II)-A/2023

Paper Code 8 1 9 6

(For All Sessions)

(Group-I)

Time: 30 Minutes

Marks : 20

Mathematics (Objective)

Rwp-12-2-23

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

- 1.1 Midpoint of A(1,2) & B(3,8) is: (A) (2, 5) (B) (4, 10) (C) (2, 6) (D) (2, 8)
2. (1, -3) is in the solution of _____ (A) $x + y \geq 1$ (B) $x + y \leq 0$ (C) $x + y = 0$ (D) $x - y = 0$
3. Centre of circle $x^2 + y^2 - 6x + 4y + 13 = 0$ (A) (3, 2) (B) (-3, 2) (C) (3, -2) (D) (-3, -2)
4. Focus of parabola $x^2 = 4ay$ is: (A) (-a, 0) (B) (0, -a) (C) (a, 0) (D) (0, a)
5. Eccentricity e for hyperbola is: (A) e = 1 (B) e = 0 (C) e < 1 (D) e > 0
6. Length of major axis of $\frac{x^2}{9} + \frac{y^2}{4} = 1$ (A) 03 (B) 06 (C) 02 (D) 04
7. Which one is not scalar quantity: (A) Work (B) Time (C) Magnetic field (D) Speed
8. $\begin{bmatrix} k & i & j \end{bmatrix}$ (A) 2 (B) 0 (C) 1 (D) -1
9. $\lim_{x \rightarrow 2} \sqrt{x^3 + 1} - \sqrt{x^2 + 5}$ (A) -1 (B) 0 (C) 2 (D) -2
10. Area of circle of unit radius is: (A) π (B) 2π (C) π^2 (D) $2\pi^2$
11. $\frac{d}{dx}(3^x) =$ _____ (A) $3^x \ln x$ (B) $3^x \ln 2$ (C) $3^x \ln 3$ (D) $x 3^{x-1}$
12. Lagrange used _____ notation for derivative. (A) $D f(x)$ (B) $f'(x)$ (C) $\frac{d}{dx} f(x)$ (D) $f(x)$
13. $\frac{d}{dx} \cos 7x =$ _____ (A) $7 \sin 7x$ (B) $-7 \sin 7x$ (C) $7 \cos 7x$ (D) $-7 \cos 7x$
14. Minimum value of function $f(x) = x^2 + 2x - 3$ is at $x =$ _____ (A) -3 (B) -2 (C) 0 (D) -1
15. $\int \frac{1}{1+x^2} dx =$ _____ (A) $\sin^{-1} x + c$ (B) $\cos^{-1} x + c$ (C) $\tan^{-1} x + c$ (D) $\cot^{-1} x + c$
16. $\int \frac{1}{x^2} dx =$ _____ (A) $-\frac{1}{x} + c$ (B) $\frac{1}{x} + c$ (C) $\frac{2}{x} + c$ (D) $-\frac{2}{x} + c$
17. Solution of $\frac{dy}{dx} = 1$ is _____ (A) $y = x^2 + c$ (B) $y = e^x + c$ (C) $y = \ln x + c$ (D) $y = x + c$
18. $\int_0^1 3x^2 dx =$ _____ (A) 3 (B) 1 (C) 2 (D) 0
19. Equation of line through origin with slope 2: (A) $2x - y = 0$ (B) $2x + y = 0$ (C) $x + 2y = 0$ (D) $x - 2y = 0$
20. Slope of line parallel to y-axis: (A) -1 (B) 0 (C) ∞ (D) 1

ematics (Subjective)

(For All Sessions)

(GROUP-II)

Time: 2:30 hours

SECTION-I

Rwp-12-2-23

Write short answers of any eight parts from the following:

(8x2=16)

- i. Express perimeter P of a square as a function of its area A.
- ii. If $f(x) = (-x + 9)^3$, find $f^{-1}(x)$
- iii. Find $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$
- iv. Differentiate w.r.t "x" $(\sqrt{x} - \frac{1}{\sqrt{x}})^2$
- v. If $y = \sqrt{x + \sqrt{x}}$ find $\frac{dy}{dx}$
- vi. Find $\frac{dy}{dx}$ if $x = y \sin y$
- vii. Find $f'(x)$ if $f(x) = x^3 \cdot e^{1/x}$
- viii. If $y = x^2 \cdot \ln(\frac{1}{x})$, find $\frac{dy}{dx}$
- ix. If $y = \sin^{-1}(\frac{x}{2})$, Find $\frac{dy}{dx}$
- x. Apply the Maclaurin series to prove that: $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$
- xi. Graph the solution set of linear inequality in xy - plane, $2x + y \leq 6$
- xii. What is a feasible solution?

3. Write short answers of any eight parts from the following:

(8x2=16)

- i. Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ for $x^2 + 2y^2 = 16$
- ii. Evaluate: $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$
- iii. Evaluate: $\int \frac{x+2}{\sqrt{x+3}} dx$
- iv. Evaluate: $\int \tan^{-1} x dx$
- v. Evaluate: $\int \frac{5x+8}{(x+3)(2x-1)} dx$
- vi. Evaluate: $\int_2^0 \frac{1}{(2x-1)^2} dx$
- vii. Solve the differential equation $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$
- viii. Find sum of \overline{AB} and \overline{CD} where $A(1, -1)$, $B(2, 0)$, $C(-1, 3)$ and $D(-2, 2)$
- ix. Find direction Cosines of vector $\underline{V} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$
- x. Find α so that $\underline{U} = 2\alpha\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\underline{V} = \mathbf{i} + \alpha\mathbf{j} + 4\mathbf{k}$ and perpendicular.
- xi. Compute $\underline{a} \times \underline{b}$ and $\underline{b} \times \underline{a}$ for $\underline{a} = \mathbf{i} + \mathbf{j}$, $\underline{b} = \mathbf{i} - \mathbf{j}$
- xii. Find volume of parallelepiped determined by $\underline{U} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\underline{V} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\underline{w} = \mathbf{i} - 7\mathbf{j} - 4\mathbf{k}$

4. Write short answers of any nine parts from the following:

(9x2=18)

- i. The point $C(-5, 3)$ is the center of the circle and $P(7, 2)$ lies on the circle. What is the radius of the circle.
- ii. Show that the points $A(0, 2)$, $B(\sqrt{3}, -1)$ and $C(0, -2)$ are vertices of a right triangle.
- iii. The points $P(-2, 6)$ and $O(-3, 2)$ are given in xy - coordinate system. Find the XY - Coordinate of P referred to the translated axes OX' and OY' .
- iv. Find an equation of the line through $(-5, -3)$ and $(9, -1)$.
- v. Convert $4x + 7y - 2 = 0$ in slope-intercept form.
- vi. Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$
- vii. Find the point of intersection of the lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$
- viii. Find center and radius of circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- ix. Find focus and vertex of parabola $y^2 = -12x$
- x. Find foci of an ellipse $9x^2 + y^2 = 18$
- xi. Find eccentricity of hyperbola, $\frac{y^2}{4} - x^2 = 1$
- xii. Write parametric equations of hyperbola.
- xiii. Write down equation of tangent to the circle $x^2 + y^2 = 25$ at $(4, 3)$.

SECTION-II

(10x3=30)

Note Attempt any three questions. Each question carries equal marks:

- 5. (a) Evaluate: $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$
- (b) Find $\frac{dy}{dx}$ if $x\sqrt{1+y} + y\sqrt{1+x} = 0$.
- 6. (a) Evaluate: $\int \frac{x}{x^4 + 2x^2 + 5} dx$
- (b) Find equation of the line through $(5, -8)$ and perpendicular to the join of $A(-15, -8)$ and $B(10, 7)$.
- 7. (a) Solve the differential equation $(y - x \frac{dy}{dx}) = 2(y^2 + \frac{dy}{dx})$
- (b) Graph the feasible region of the following system of linear inequalities and find the corner points.
 $2x + y \leq 10, x + 4y \leq 12, x + 2y \leq 10, x \geq 0, y \geq 0$
- 8. (a) Show that $y = \frac{\ln x}{x}$ has maximum value at $x = e$.
- (b) Write an equation of the circle that passes through the given points $A(4, 5)$, $B(-4, -3)$, $C(8, -3)$
- 9. (a) Find the focus, vertex and directrix of the parabola $x^2 - 4x - 8y + 4 = 0$